Current controlled current source model for a PWM dc-dc boost converters operated in discontinuous current mode2000 IEEE

ISCAS 2000 - IEEE International Symposium on Circuits and Systems, May 28-31, 2000, Geneva, Switzerland

CurrentControlled CurrenSource Model for a PWM DCDC Boost Converters operated in Discontinuos Current Mode

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Abstract - A small-signal model for a PWM dc-dc boost converter operated in DCM composed of current–controlled current sources and derived by using the energy conservation approach is presented. The proposed model is suitable for small-signal, frequency-domain representation of the converter and can be used to derive the expressions and Bode plots of the control-to-output voltage transfer function, the input-to-output voltage transfer function, the input impedance, and the output impedance.

Main advantages of the proposed method are as follows.

- A linear equivalent circuit of the converter is derived and its representation in the frequency domain does not require matrix manipulation.
- The converter parasitic components are considered.
- A good understanding of the frequency domain behavior of power converters is achieved.

Finally, the proposed method can be easily utilized to model other PWM dc-dc converter circuits.

1. Introduction

Pulse width modulation (PWM) dc-dc converters are circuits based on a controlled switch and a diode cyclically switching and driving the entire converter circuit through several topological configurations constituted by linear reactive and resistive components, connected to a dc voltage source. The two configurations during one switching period for a converter operated under continuos current mode (CCM) increase to three for converters operated under discontinuous current mode (DCM).

A cycle-by-cycle simulation of switching converters is practical for time domain simulations, while a frequency domain representation takes advantage from an averaged modeling [1] and [2].

Substitution of the switching part of the converter circuits with an equivalent, averaged, linear, time invariant, linearized equivalent circuit has been proposed in [3]-[8]. This model is utilized for numerical frequency-domain simulations of dc-dc converters by using analog simulators like SPICE and/or mathematical simulators like MATLAB. A systematic method for including parasitic components into static and dynamic models of PWM converters operated in CCM has been presented in [9]. By using this methods s-domain the transfer functions describing the converter behavior are derived and parasitic resistances of the converter components are taken into account. As a result, equivalent circuits suitable for an appropriate design of the feedback network are obtained [10]–[12].

The purpose of this paper is to present a linear model of a dc-dc PWM boost converters operated in DCM. Parasitic resistances of the converter components are considered by using the energy conservation approach [9]. The model is suitable to derive the expressions for the control-to-output voltage transfer function, the input-to-output voltage transfer function, the input impedance, and the output impedance.

2. The CurrentControlled CurrentSource Model

Fig. 1(a) shows the "switching block" constituted of a controlled switch S and a diode D and its connection to the inductor L. This sub-circuit is the "core" part of a dc-dc switching converter. As shown in Fig. 1(b) the combination of the controlled switch S and diode D acts as a device diverting the inductor current i_L through the switch when ON and through the diode when S is OFF.

Fig. 2 shows the waveforms of inductor, switch and diode current and inductor voltage. For a converter operated in DCM the inductor current is zero from time D_1T to the end of the switching period T. During this time interval both S and D are OFF.

The voltage across the inductor is

$$V_L = \begin{cases} L\dfrac{i_{\max}}{DT} = V_{SL} & \text{for } 0 \le t < DT \\ -L\dfrac{i_{\max}}{(D_1 - D)T} = -V_{LD} & \text{for } DT \le t < D_1T \\ 0 & \text{for } D_1T \le t < T \end{cases} \tag{1}$$

This gives

$$V_{LD} = \frac{D}{D_1 - D} V_{SL} = \frac{1}{(\mu - 1)} V_{SL} \tag{2}$$

where $\mu = D/D_1$.

The maximum current through the components of the sub-circuit shown in Fig. 1 is

$$I_{\max} = \frac{V_{SL}}{L} DT \tag{3}$$

The average currents through branches of Fig. 1 circuit are $I_S = DI_{max}/2$, $I_D = (D_1\text{-}D)I_{max}/2$, and $I_L = D_1I_{max}/2$. Combination of these gives

$$I_S = \frac{D}{D_1} I_L = \mu\, I_L \tag{4}$$

and

$$I_D = \frac{D_1 - D}{D_1} I_L = 1 - \frac{D}{D_1} I_L = (1-\mu)\, I_L \tag{5}$$

which are the constitutive equations of the current–controlled current-sources modeling the DC behavior of switching sub-circuit, as shown in Fig. 3 (a).

Substitution of (3) into (4) gives

$$I_S = \frac{D^2}{2L} T\, V_{SL} \tag{6}$$

By considering both the DC and small-signal ac components of current I_S, duty cycle D, and voltage V_{SL} we have

$$I_S + i_S = \frac{T}{2L}(D+d)^2\,(V_{SL} + v_{SL}) \tag{7}$$

Neglecting higher order terms, the ac components of the switch branch current is

$$i_S \approx \frac{TD}{L} V_{SL} d + \frac{TD^2}{2L} v_{SL} = k_i d + g_i v_{SL} \tag{8}$$

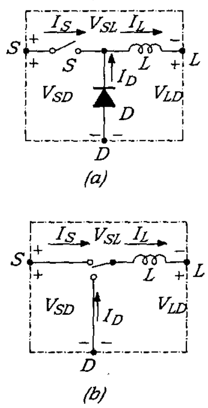

Fig. 1. Switching circuit of a PWM converter.
(a) Combination of switch, diode, and inductor .
(b) Equivalent circuit constituted by inductor and "Current diverting device".

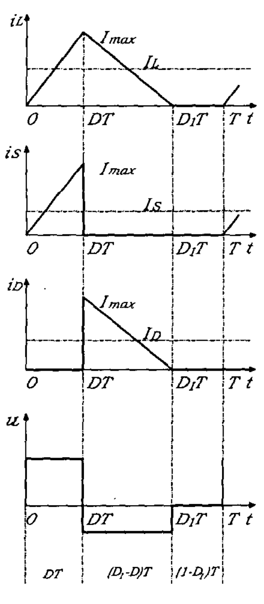

Fig. 2. Current through components of Fig. 1 and inductor voltage.

Combination of (2) and (5) gives

$$I_D = \frac{D^2 T V_{SL}^2}{2 L V_{LD}} \tag{9}$$

By considering both the DC and small-signal ac components of current I_D, duty cycle D and voltages V_{SL} and V_{LD} we have

$$I_D + i_d = \frac{T}{2L}\left[\frac{(D+d)^2 (V_{SL}+v_{SL})^2}{V_{LD}+v_{LD}}\right] \tag{10}$$

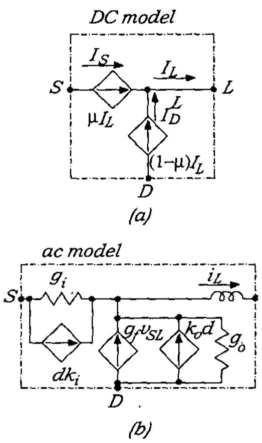

Fig. 3. Linearized equivalent circuit of the switching circuit of a PWM converter. (a) Dc model. (b) Ac model.

Neglecting higher order terms, the ac components of the diode branch current is

$$\begin{aligned} i_d &\approx \frac{TDV_{SL}}{LV_{LD}} d + \frac{TD^2 V_{SL}}{LV_{LD}} v_{SL} - \frac{TD^2 V_{SL}}{2LV_{LD}} v_{LD} \\ &= k_O d + g_f v_{SL} - g_f v_{LD} \end{aligned} \tag{11}$$

The equivalent circuit related to expressions (6) and (11) is the ac model of the switching sub-circuit shown in Fig. 3 (b).

From plots shown in Fig. 2, the expression of the switch rms current is derived as follows

$$I_{Srms} = I_L \left(\frac{4D}{3D_1^2}\right)^{1/2} = I_L \frac{V_O - V_I}{V_O}\left(\frac{4}{3D}\right)^{1/2} \tag{12}$$

From (12) the power loss in the switch ON resistance R_{DS} is calculated

$$P_S = R_{DS} I_{Srms}^2 = R_{DS} \frac{4}{3D}\left(\frac{V_O - V_I}{V_O}\right)^2 I_L^2 \tag{13}$$

According to the energy conservation method [9] the expression of the switch equivalent resistance series connected to inductor L is

$$R_{DSE} = \frac{4}{3D}\left(\frac{V_O - V_I}{V_O}\right)^2 R_{DS} \tag{14}$$

The diode rms current is expressed as

$$I_{Drms} = I_L \frac{1}{(D_1 - D)D}\left(\frac{K_1 + K_2 + K_3}{3}\right)^{1/2} \tag{15}$$

where

$$K_1 = (D_1 - D)^3 \left(16 - 3D_1^2 - D_1\right)^3 \quad K_2 = 3D_1^2 (1-D)(D_1 - D)^2$$
$$K_3 = (D_1 - D)\times \left[D_1^2\left(6D_1^2 + 24D - 12\right) + 6D\left(DD_1 - 2D - 2D_1^3\right)\right] \tag{16}$$

The power loss due to the diode forward resistance R_F is

$$P_{RF} = R_F I_{Drms}^2 = R_F \frac{K_1 + K_2 + K_3}{3}\left(\frac{1}{(D_1 - D)D}\right)^2 I_L^2 \tag{17}$$

and the diode equivalent resistance series connected to inductor L is expressed as

$$R_{FE} = R_F \frac{K_1 + K_2 + K_3}{3}\left(\frac{1}{(D_1 - D)D}\right)^2 \tag{18}$$

Combination of (14), (18) yields the expression of inductor equivalent series resistance

$$r = R_L + R_{DSE} + R_{FE} = R_L + \left(\frac{V_O - V_I}{V_O}\right)^2 \frac{4}{3D} R_{DS} + \\ + \frac{K_1 + K_2 + K_3}{3}\left(\frac{1}{(D_1 - D)D}\right)^2 R_F \tag{19}$$

The power loss due to the diode threshold voltage V_F is

$$P_{VF} = V_F I_D = V_F \frac{D_1 - D}{D_1} I_L, \tag{20}$$

which allows the equivalent voltage series connected to inductor L to be calculated as

$$V_{FE} = V_F\left(1 - \frac{D}{D_1}\right) = V_F\left(\frac{1}{\mu} - 1\right) = V_F \frac{V_I}{V_O} \tag{21}$$

The schematic circuit of a PWM boost converter is shown in Fig. 4(a). Figs. 4(b) and (c) shows, the converter linearized equivalent dc and ac circuits, respectively.

From the dc equivalent circuit the dc output voltage expression is derived

$$V_O = \frac{\frac{1}{1-\mu} V_I - \frac{1}{\mu} V_F}{1 + r/R(1-\mu)} \tag{22}$$

By neglecting the diode threshold voltage the steady state voltage transfer function of the boost converter, including the parasitic components can be expressed in a closed form as

$$M_{VDC} \equiv \frac{V_O}{V_I} = \frac{1}{1-\mu} \times \frac{1}{1 + r/R(1-\mu)} \tag{23}$$

For an ideal converter $r = 0$ and

$$M^{I}_{VDC} = \frac{1}{1-\mu} = \frac{1}{1 - \frac{D}{D_1}} \tag{24}$$

which is also the expression for the voltage transfer function derivable from the circuit shown in Fig. 4(b) by setting $V_F = 0$ and $r = 0$.

Note that the for a converter operated with a inductor current reaching zero at $t = T$, we have $D_1 = 1$. Substitution of this into (23) yields

$$M^{II}_{VDC} = \frac{1}{1 - D + r/R} \tag{25}$$

which is the voltage transfer function of a real boost converter operated in CCM. For an ideal converter $r = 0$ and (23) modifies to the expression $M = 1/1\text{-}D$ normally used when an ideal boost converter is assumed to be operated in CCM.

The current transfer function is

$$M_{IDC} \equiv \frac{I_O}{I_I} = 1 - \mu \tag{26}$$

Combination of this and (23) allows the converter efficiency to be calculated as

$$\eta_{DC} \equiv \frac{P_O}{P_I} = \frac{V_O I_O}{V_I I_I} = \frac{1}{1 + \frac{r}{R(1-\mu)}} \tag{27}$$

The control-to-output transfer function of the boost converter operated in DCM is derived from the circuit shown in Fig. 4(c) with $v_i = 0$ as

$$T_{PDCM}(s) = \left.\frac{v_O(s)}{d(s)}\right|_{v_i=0} = \frac{a_2 s^2 + a_1 s + a_0}{b_2 s^2 + b_1 s + b_0} \tag{28}$$

where

$$a_2 = LCRR_C\left[k_i(g_f + g_O) - k_O g_i\right] \tag{29}$$

$$a_1 = LR\left[k_i(g_f + g_O) - k_O g_i\right] + \\ + CRR_C\left[r k_i(g_f + g_O) - r k_O g_i - k_O\right] \tag{30}$$

$$a_0 = rR\left[k_i(g_f + g_O) - k_O g_i\right] - k_O R \tag{31}$$

$$b_2 = LCRR_C g_i g_O + LC(R + R_C)(g_i + g_f + g_O) \tag{32}$$

$$b_1 = L\left[g_O(R g_i + 1) + (g_f + g_i)\right] + CRR_C g_O(r g_i + 1) + \\ + C(R + R_C)\left[r(g_f + g_i + g_O) + 1\right] \tag{33}$$

$$b_0 = R g_O(r g_i + 1) + r(g_f + g_i + g_O) + 1. \tag{34}$$

Coefficients a_0, a_1, a_2, can be negative, this demonstrates the potential open-loop unstable operation of the boost converter.

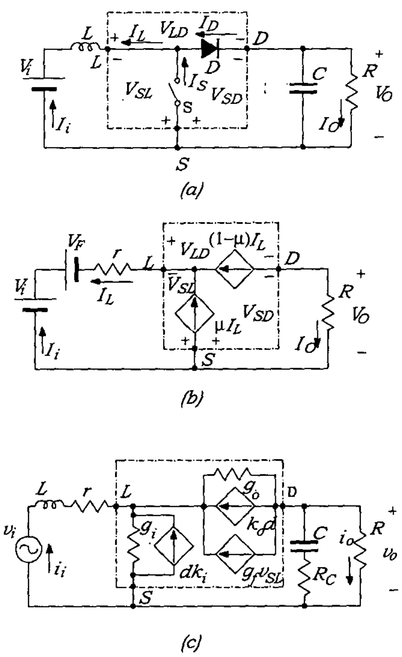

Fig. 4. PWM boost converter.
(a) Schematic circuit. (b) Equivalent linearized DC circuit.
(c) Equivalent linearized ac circuit.

The input-to-output voltage transfer function is derived by the circuit shown in Fig. 4 (c) with $d = 0$ as

$$M_{VaDCM}(s) = \left.\frac{v_O(s)}{v_I(s)}\right|_{d=0} = H_V \frac{\alpha_1 s + 1}{\beta_2 s^2 + \beta_1 s + b_0} \tag{35}$$

where

$$H_V = R(g_f + g_O) \tag{36}$$

$$\alpha_1 = CR_C \tag{37}$$

$$\beta_2 = LC\left[RR_C g_i g_O + (R + R_C)(g_f + g_O)\right] \tag{38}$$

$$\beta_1 = R g_O\left[L g_i + CR_C(r g_i + 1)\right] + \\ + (g_f + g_O)\left[L + rC(R + R_C)\right] \tag{39}$$

$$\beta_0 = Rg_O(g_i + 1) + r(g_f + g_O). \tag{40}$$

The expressions of the input and output impedance can be similarly derived.

3. Application Example

A PWM boost dc-dc DCM with a switching frequency $f = 50$ kHz, a dc input voltage $V_{IN} = 10$ V, a 40 W output power and a nominal output voltage $V_O = 20$ V is considered as an example. The converter nominal duty cycle is $D = 0.158$, $D_1 = 0.316$, and a load resistance $R = 10\ \Omega$. The converter inductance is $L = 10\ \mu$H and the output capacitance is $C = 200\ \mu$F. The parasitic components of the converter equivalent circuit are $R_L = 100$ mΩ; $R_F = 25$ mΩ; $R_{DS} = 20$ mΩ.

The plots of magnitude and phase of control-to-output voltage transfer function are shown in Fig. 5.

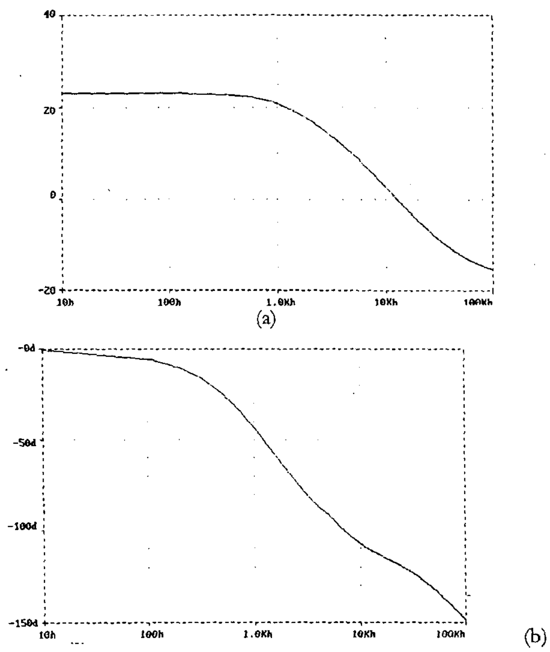

Fig. 5. Bode plots of the boost converter considered as an application example.(a) Magnitude (dB). (b) Phase (Degree)

4. Conclusions

Using the energy conservation approach has derived a linear model of a dc-dc PWM boost converter operated in DCM constituted of current-controlled current-sources.

This model allows us to determine the expressions for the control-to-output voltage transfer function, input-to-output voltage transfer function, input impedance, and the output impedance expressions and their Bode plots.

The proposed models results in the following advantages:

- the converter transfer function expressions include all parasitic components in the converter circuit and can be reduced by neglecting some of them.
- The circuit model can be used in numerical circuit simulators like SPICE, it is suitable for symbolic analysis algorithms, and the transfer function expressions can be processed by using any general purpose numerical and/or symbolic program like MATLAB, MATHCAD, SCIENTIFIC WORKPLACE etc.
- The current-controlled current source model of PWM the dc-dc boost converter allows for a good understanding of the frequency domain behavior of converter circuit, and can be usefully applied to the design of the compensating networks used in the converter feedback loop.

References

[1] S. Ben-Yaacov, D. Edry, "Averaged models and tools for studying the dynamics os switch mode DC-DC converters," Proceedings of IEEE Power Electronics Specialist Conference PESC 1994, Taipei, 1994, pp. 1218-1225.

[2] A. Luchetta, S. Manetti, M.C. Piccirilli, A. Reatti, "Frequency Domain Analysis of DC-DC Converters Using a Symbolic Approach", Proceedigns of ISCAS'95, International Symposium on Circuit and Systems, Special Session on Circuit Theory Aspects in Power Electronics, Seattle, WA, April 29 - May 3, 1995, pp. 2043-2046.

[3] V. Vorpérian, "Simplified analysis of PWM converters using model of PWM switch. Part I: Continuous Conduction Mode," IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, No. 3, May 1990, pp. 490-496.

[4] V. Vorpérian, "Simplified analysis of PWM converters using model of PWM switch. Part II: Discontinuous Conduction Mode," IEEE Transactions on Aerospace and Electronic Systems, Vol. 26, No. 3, May 1990, pp. 497-505.

[5] B. Ridley, "A new continuous model for current mode control," IEEE Transactions on Power Electronics, Vol. 6, No. 2, April 1991, pp. 339-346.

[6] Y. Amran, F. Huliehel, S. Ben-Yaacov, "A unified SPICE compatible average model of PWM converters," IEEE Transactions on Power Electronics, Vol. 6, October 1991, pp. 585-594.

[7] D. Edry, S. Ben-Yaacov, "Averaged simulation of PWM converters by direct implementation of behavioral relationship," International Journal of Electronics, Vol. 77, 1994, pp. 731-746.

[8] S. Ben-Yaacov, "Behavioral average modeling and SPICE simulation of PWM and resonant converters," Copyright 1998@S. Ben-Yaakov.

[9] D. Czarkowski and M. K. Kazimierczuk, "Energy-conservation approach to modeling PWM DC-DC Converters," IEEE Transactions on AES Vol 29, no. 3 July 1993.

[10] M. K. Kazimierczuk, R.C. Cravens, A. Reatti, "Closed-Loop Input impedance of PWM Buck-Boost DC-DC Converters", Proceedings of ISCAS'94, International Symposium on Circuit and Systems, London, England, May 30-June 2, 1994, Vol. 6, pp. 61-64.

[11] M. K. Kazimierczuk, R.S. Geise, A. Reatti, "Small Signal Analysis of a PWM DC-DC Converter with A Non-Symmetric Integral-Lead Controller", Proceedings of Intelec'95-International Telecommunications Energy Conference, The Hague, The Netherlands, October 29-November 1, 1995, pp. 608-615.

[12] M. Bartoli, A. Reatti, M. K. Kazimierczuk, "Open loop small-signal control-to-output transfer function of PWM buck converter for CCM: modeling and measurements", Proceedings of MELECON'96, Bari, Italy, May 13-16, 1996, pp. 1203-1206.